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STEADY STATE OSCILLATIONS
IN
SWITCHING REGULATED DC TO DC
POWER SUPPLIES

BY
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B.E., University of Bombay, 1959

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Engineering
in the Graduate Studies program of
Florida Technological University

Orlando, Florida

1974

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INTRODUCTION

Regulation by Switching

The switching regulated power supply has gained considerable popularity and acceptance in applications requiring sources of power that can provide regulated dc voltages. The switching regulator differs from the conventional series regulator in its basic approach. In the series regulator, a resistance in series with the unregulated dc voltage is varied so that the excess voltage is dropped across it and only the required voltage appears at the output terminals. This results in a considerable power loss in the series resistor and hence the series regulator has a low overall efficiency. On the other hand, the switching regulator continually switches the unregulated power on and off in such a manner that a desired voltage is obtained at the output terminals. The output voltage thus obtained, has a high ripple content which is reduced to acceptable levels by filtering. The switching operation takes place at frequencies in the range of 10 kHz. to 40 kHz. The switching technique offers a theoretical efficiency of 100 percent, although practical units have efficiencies in the range

of 90 to 95 percent.

Apart from the improved power efficiency, the switching regulator results in a more compact unit compared to a series regulated unit. This is so because the reduction in power loss reduces the need for heat sinks and ventilation fans. As a result switching regulated power supplies are more efficient and compact compared to the series regulated power supplies.

One important class of the switching regulated power supplies is the dc to dc step down power supply where the output voltage is lower than the input voltage under all operating conditions. This is the type of unit that finds applications in computers, business machines and industrial equipment where the basic dc power is obtained by rectifying the power from ac supply lines and the power supply is used for getting a lower, regulated dc voltage at the output. Also in some cases where dc power is available, and a lower dc voltage may be desired, it can be accomplished by this class of switching regulators.

Specifications

The basic specification of a switching regulated power supply is given in terms of the input and output requirements. The input requirements are specified as the nominal dc input voltage and the range over which it is likely to vary in operation. The output requirements are

specified in terms of the desired dc output voltage, the required voltage regulation under a specified range of the load current and the allowable peak to peak ripple in the output voltage.

Operating Principles

Figure 1 in chapter 1 shows a simple circuit arrangement of a switching regulated dc to dc power supply, suitable for step down voltage applications. The transistor (Q) operates as a switch that is driven into the on state, or into the off state by the driver circuit. The driver circuit receives its signal from the voltage comparator that determines the value of $(v_0 - V_R)$, where v_0 is the output voltage and V_R is the reference voltage. The driver circuit itself is a triggering circuit such as an IC voltage regulator. The diode (D) provides a path for the load current when the transistor (Q) is not conducting. The combination of inductor (L) and capacitor (C) provides the filtered output voltage (v_0) across the load resistor (R).

To start with, assume that the transistor (Q) is driven into the on state and a voltage equal to the input voltage less the transistor voltage drop, appears across the diode (D). This voltage causes reverse biasing of the diode which remains in the off state. The dc input voltage (V_i) will then charge the capacitor (C) via the transistor (Q) and the inductor (L) and at the same time feed the load

resistor (R). As the capacitor is charged, the output voltage (v_0) will rise until it reaches a predetermined value such that

$$(v_0 - V_R) = \Delta_1,$$

where Δ_1 is the upper switching voltage for the driver circuit.

As soon as the above state is reached, the driver circuit will cause the transistor (Q) to switch to the off state and stop the flow of the current from the source into the circuit. At this stage, the inductor (L) will not allow a sudden interruption in current and it continues to flow through the diode (D) which is now forward biased. The energy storage elements, inductor (L) and capacitor (C) will continue to feed the load current during this period. As the capacitor discharges, the output voltage v_0 will start falling, until it reaches a predetermined value such that

$$(v_0 - V_R) = -\Delta_2,$$

where $-\Delta_2$ is the lower switching voltage for the driver circuit.

At this point, the driver circuit will cause the transistor (Q) to be driven into the on state and start up the initial cycle of rising voltage output, once again. This cycle of rising and falling voltages is repeated continuously.

The Approach

The voltage ripple appearing in the output voltage of the dc to dc power supply, has a certain magnitude and a certain frequency of oscillation. These characteristics depend on circuit parameters and the input voltage. In the first chapter, the author presents a basic problem in this area. An analytical method of solution, utilizing nonlinear control systems theory is discussed. It is supported with a comparison of test data on a laboratory model and the calculations using the analytical method. In the following chapters additional information regarding various details of procedures, tests and calculations is provided.

CHAPTER 1

STEADY STATE OSCILLATIONS IN SWITCHING REGULATED DC TO DC POWER SUPPLIES

Introductory

A switching regulated dc to dc power supply is characterized by periodic switching on and off of the input power into a filter circuit. The output from the filter circuit contains an ac ripple having the same fundamental frequency as that of the periodic switching. This frequency of oscillation is a function of the various circuit parameters, the input voltage, and the load current. When a complete range of the input voltages and the load currents is specified, it is desirable to know the frequencies of oscillation over the entire spectrum.

The author presents here a method of determining the oscillatory frequency, based on the technique of dual-input describing function discussed by Hsu and Meyer (1968). The approach allows division of the circuit into linear and nonlinear sections which can be represented separately. The transfer functions of the various sections are then combined into a control systems model which is solved to determine the oscillatory frequency.

The method of analysis presented here is graphical and provides insight into the circuit operation. The theoretical approach is supported by a comparison between the analytical results and the test results on a laboratory model. Various other implications of the graphical analysis are also discussed. Finally, the author compares the approach here with the techniques adopted by others working in the same field and shows several advantages.

The Basic Assumption

The dual-input describing function technique is an extension of the describing function procedure. In the describing function approach, there is one fundamental assumption made. That is, although the output from a nonlinear circuit may contain a fundamental ac component together with several higher order harmonics, the fundamental component alone is considered for analysis. All the components of higher order are neglected. This assumption simplifies the calculations greatly. It is valid when a nonlinear circuit is connected to a low-pass filter at its output. As will be shown later, such a situation does exist in the type of circuits we are considering. Of course, how effective a filter is in attenuating the harmonic components will depend on the design of the filter, and will vary from one circuit to

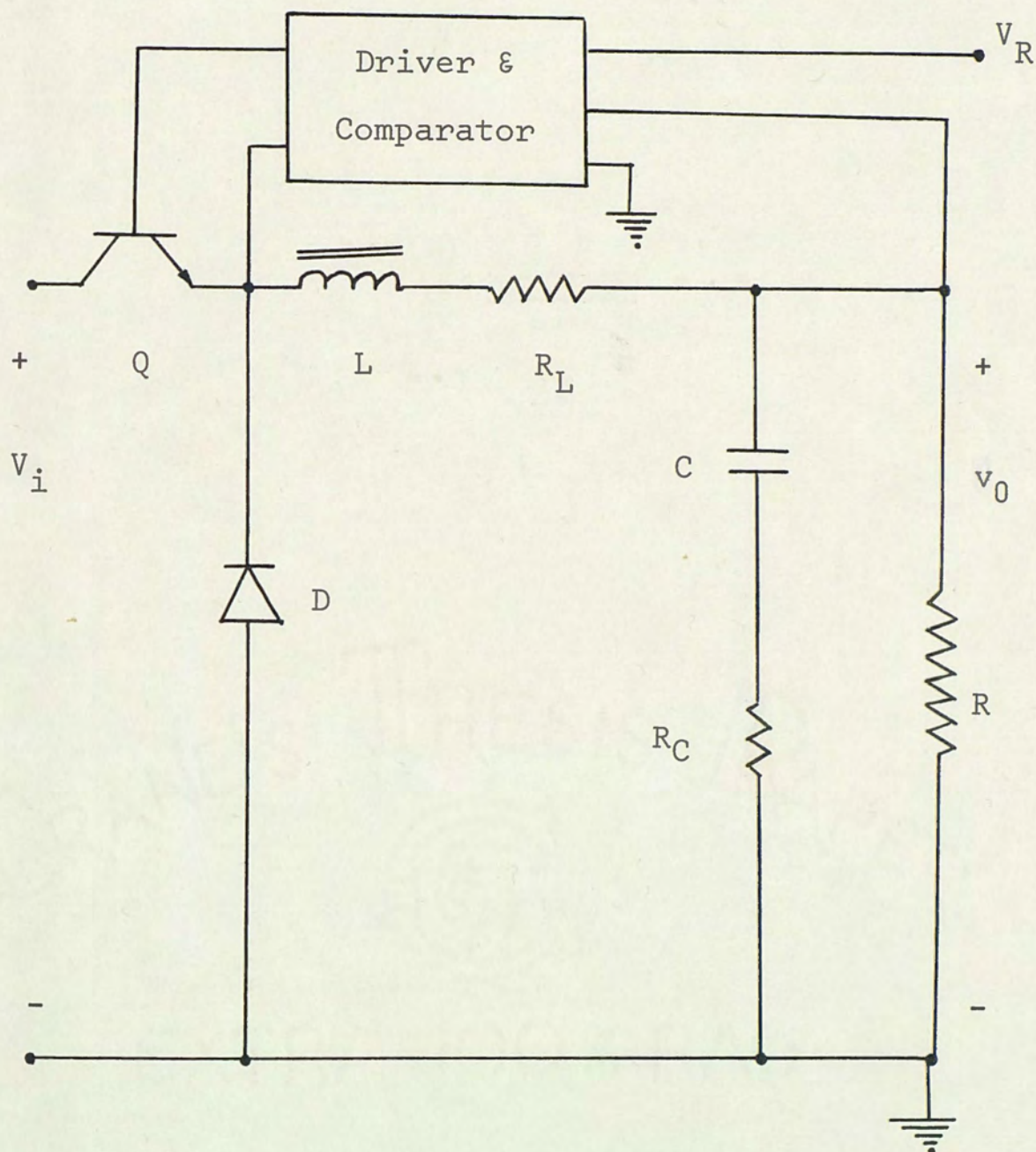
another. In general, for most circuits the assumption is valid and will provide fairly accurate results. This is particularly true with the calculation of frequency of oscillation as will be shown later.

The describing function approach requires that the nonlinearity should produce a symmetrical output and allow no dc component in its output. This is not true for the circuits that we shall consider. In fact, the basic output of interest from a dc power supply, is its dc output. In the dual-input describing function approach, this difficulty is resolved by taking two outputs, namely, the dc output and the fundamental ac output into consideration.

Circuit Model

Figure 1 represents the basic elements of a step down switching regulated dc to dc power supply. V_i is the available dc voltage from which the load resistor (R) receives a regulated output voltage (v_o). The output voltage (v_o) is compared with a reference voltage (V_R) and the driver circuit provides a switching signal to the transistor (Q), depending on the value of ($V_R - v_o$). Inductor (L) and capacitor (C) act as the low-pass filter discussed earlier. The diode (D) provides an alternate path for the load current when the transistor (Q) is turned off. It should be noted that when the transistor is turned on, it is driven into saturation and has only a small voltage drop across it.

Figure 1
The Basic Circuit



The circuit shown in figure 1 can be divided into the following sections:

i) The nonlinear section comprising the switching transistor (Q) and the diode (D)

ii) The linear section comprising the inductor (L), the capacitor (C), and the load resistor (R)

iii) The linear time delay function associated with the driver circuit, the comparator, and the switching transistor.

If each section is modeled as a transfer function, the entire circuit can be represented as a control system in terms of the transfer functions for each of the sections.

The nonlinear circuit can be looked upon as a block shown in figure 2, with an input of $(V_R - v_0)$. When this input voltage is rising and reaches the value equal to the upper switching voltage $(+\Delta_1)$, the transistor (Q) is driven into the on state and the output voltage A is given by:

$$A = V_i - V_Q$$

where V_i = input source voltage

and V_Q = transistor voltage drop.

On the other hand, when the input to the block is falling and reaches the lower switching voltage $(-\Delta_2)$, the transistor is driven into the off state. The output of the nonlinear block equals the diode forward drop because the diode is now conducting. This is shown as the voltage of -B. The block

Figure 2

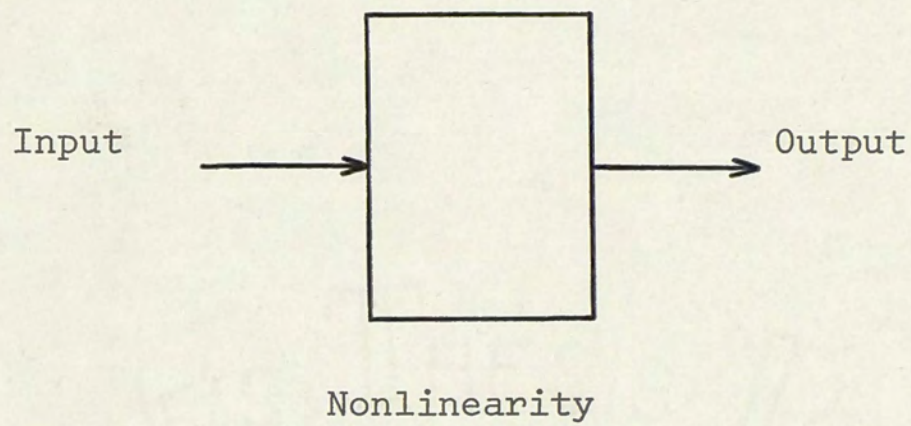
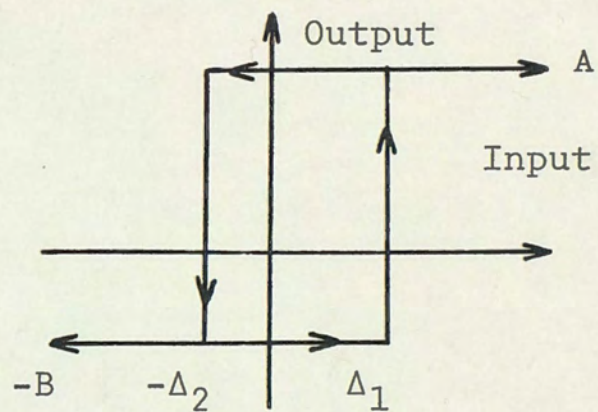
Nonlinear Section

Figure 3

Hysteresis Loop

can now be seen as the hysteresis loop shown in figure 3. It is not symmetrical with respect to the horizontal axis, showing that a dc output as discussed earlier, does exist.

The linear section is comprised of the inductor (L) and associated resistance (R_L), the capacitor (C) and associated series resistance (R_C), and the load resistance (R). This can be represented as a voltage transfer function $G(s)$ given by the following expression:

$$(1-1) \quad G(s) = \frac{R+s(CRR_C)}{LC(R+R_C)s^2 + \{L+CRR_C+CR_L(R+R_C)\}s + R+R_L}$$

Complete derivation of this expression is given in chapter 2.

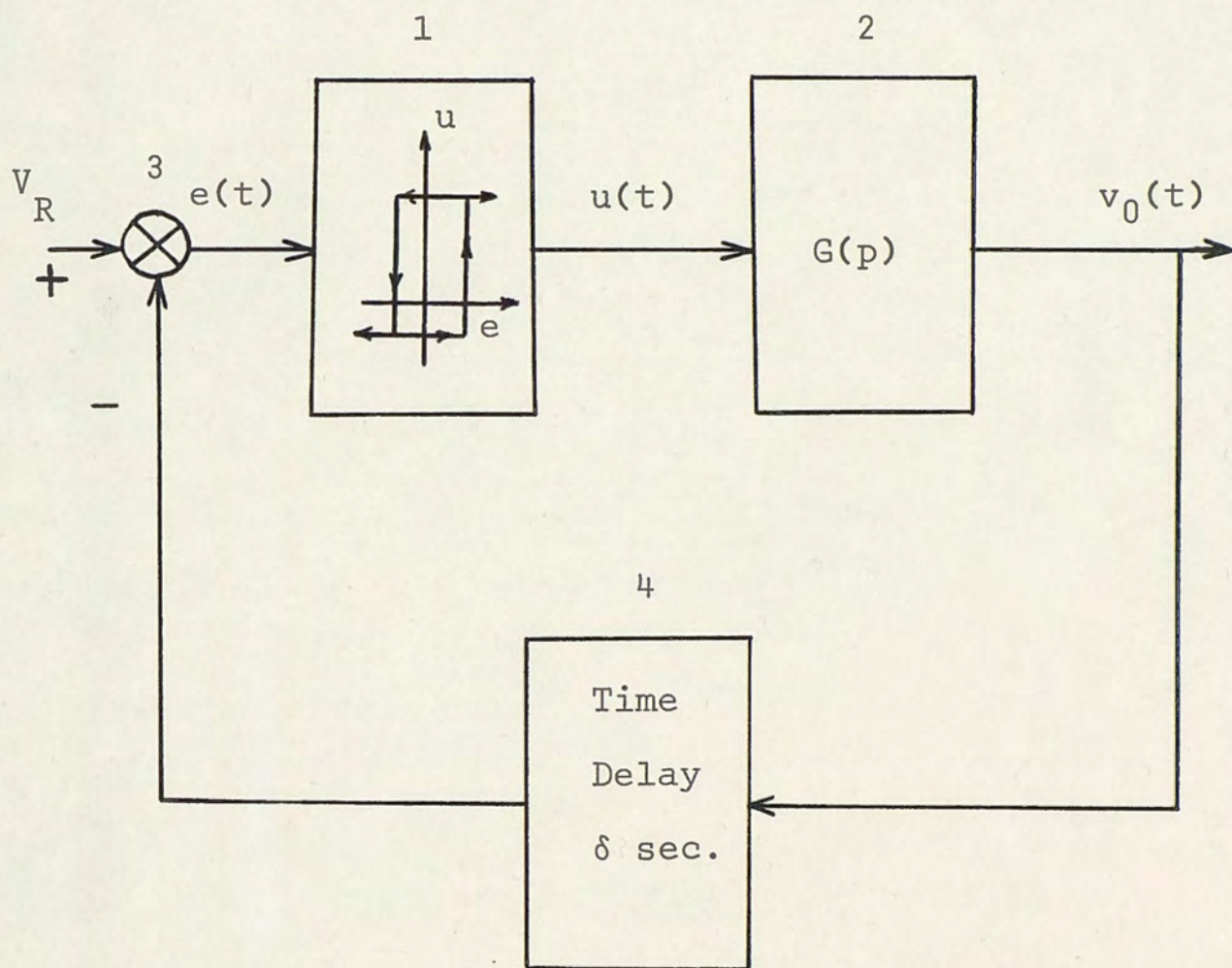
There is a certain amount of time delay that occurs when the driver circuit responds to a signal. Also, the switching operation of the transistor (Q) has a time delay associated with it. The two time delay elements are considered as lumped into one circuit section having a total time delay equal to δ .

All the circuit elements are now defined and the entire circuit can be represented as a control system shown in figure 4.

Application of the Dual-Input Describing Function Technique

The basic principle of dual-input describing function (DIDF) technique is discussed here in brief. In the DIDF

Figure 4

Control System Representation

- 1 The nonlinear section.
- 2 The linear section.
- 3 The driver and comparator.
- 4 The time delay function.

Note: $p = d/dt$; $v_0(t) = G(p)u(t)$ is the differential equation of the linear section.

approach, we define $e(t) = \alpha + \beta \sin \omega t$ because we are interested in the dc and the fundamental ac components. Figure 5 shows the control system model in terms of the transfer functions related to the dc and fundamental ac components of the output. $G(j\omega)$ and $D(j\omega)$ are the transfer functions of the linear circuit and the time delay function expressed in frequency domain. $G(0)$ and $D(0)$ are the corresponding transfer functions for the dc model and $G_1(j\omega)$, $D_1(j\omega)$ are corresponding transfer functions for the ac model. We define $N_0(\alpha, \beta)$ as the dc describing function and $N_1(\alpha, \beta)$ as the first harmonic describing function for the nonlinearity. Both $N_0(\alpha, \beta)$ and $N_1(\alpha, \beta)$ can be determined by Fourier series analysis, using figure 6. The value of $N_0(\alpha, \beta)$ varies considerably depending on whether α is positive or negative and whether its magnitude is greater or smaller than the switching voltages Δ_1 and Δ_2 . In chapter 2, page 47, it is shown that the upper and lower input switching voltages for the hysteresis nonlinearity can be made symmetrical by defining V_R' in the center of the switching band. With this choice for reference, the switching limits can be written as $+\Delta$ and $-\Delta$ as shown in figure 6. Values of $N_0(\alpha, \beta)$ under various conditions are shown in table 1. $N_1(\alpha, \beta)$ is given by

$$(1-2) \quad N_1(\alpha, \beta) = (A+B)(\cos \omega t_1 + \cos \omega t_2 - j2\Delta/\beta)/\pi\beta$$

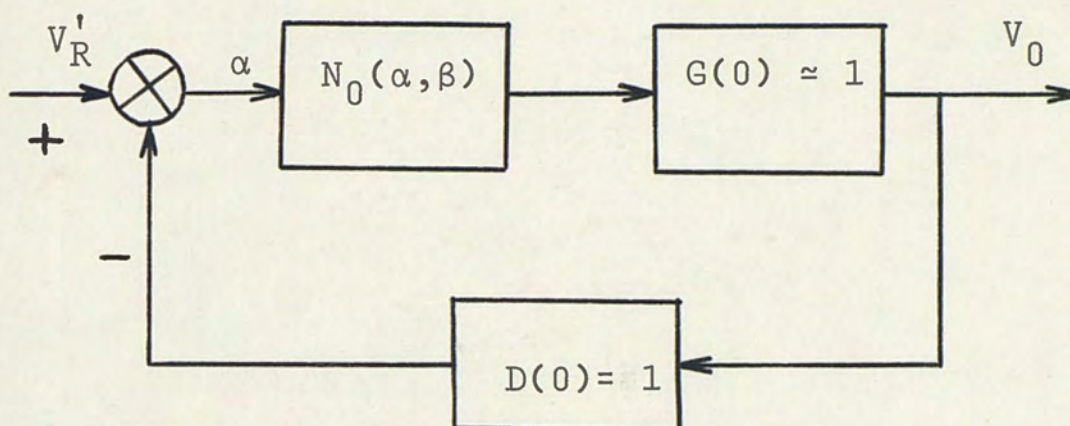
$$\text{where} \quad \cos \omega t_1 = \left\{ 1 - \frac{(\Delta - \alpha)^2}{\beta^2} \right\}^{\frac{1}{2}}$$

$$\text{and} \quad \cos \omega t_2 = \left\{ 1 - \frac{(\Delta + \alpha)^2}{\beta^2} \right\}^{\frac{1}{2}}$$

Figure 5

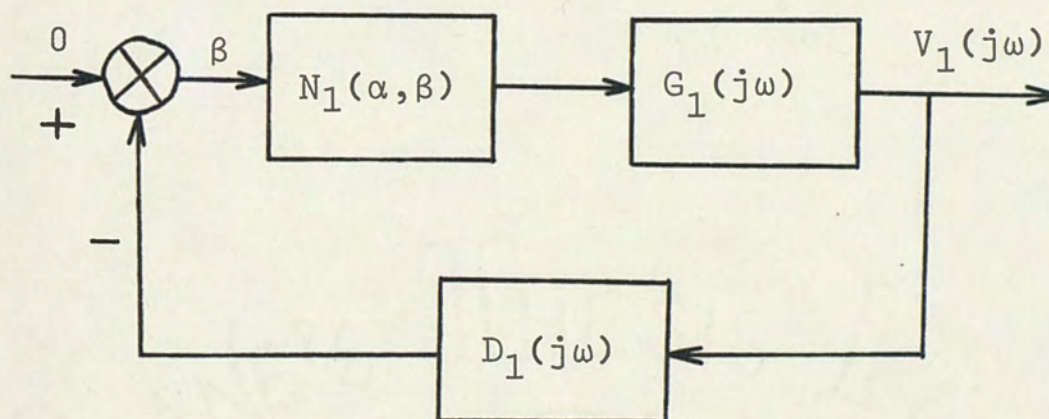
Control System Model

I. For the dc component of the model.



Note: V_0 is the dc component of the output voltage.

II. For the fundamental ac component of the model.



Note: $V_1(j\omega)$ is the fundamental ac component phasor of the output voltage.

Figure 6

Output From Nonlinear Section

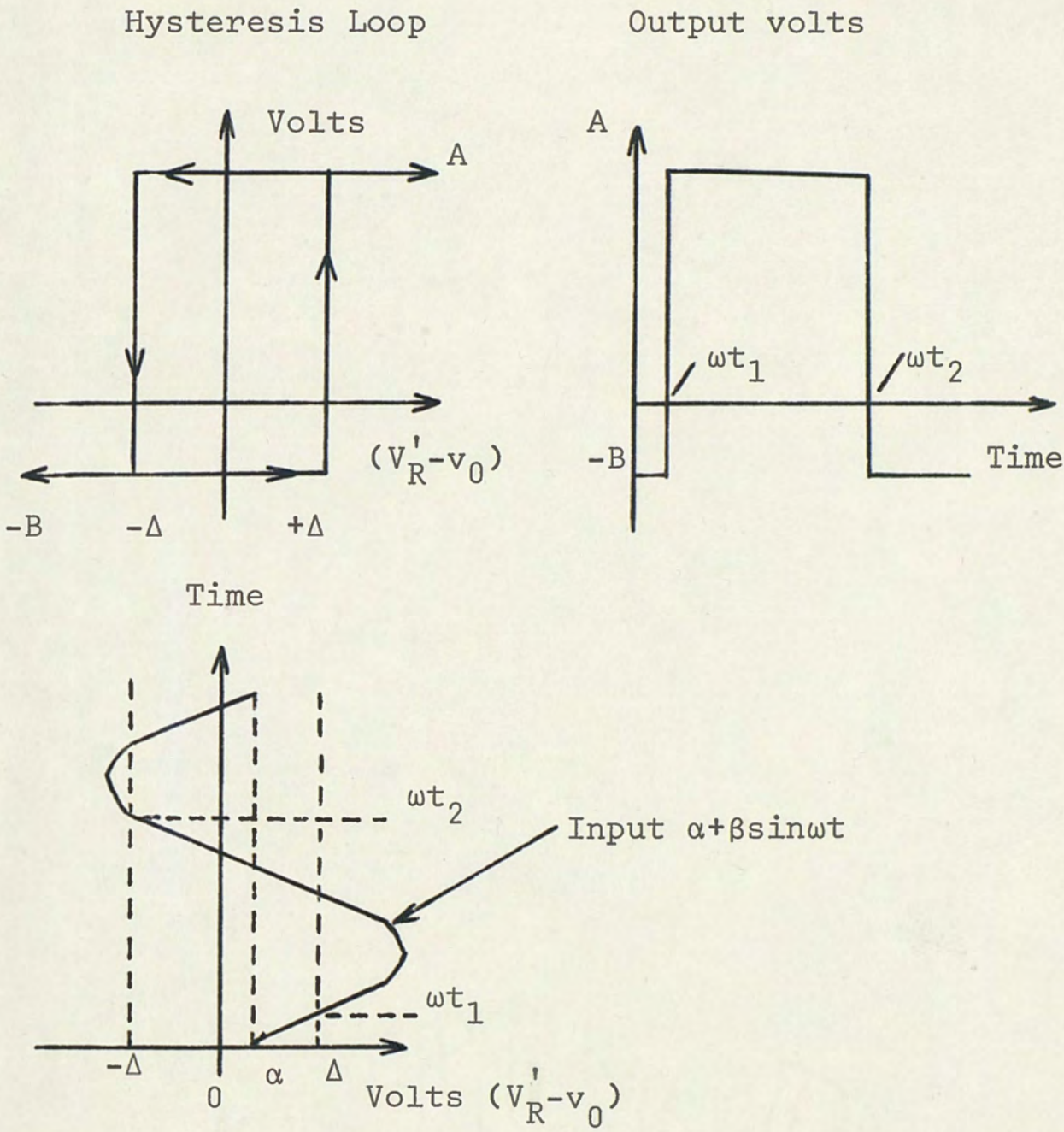


TABLE 1

 $N_0(\alpha, \beta)$ under Different Values of α

α	$ \alpha : \Delta $	$N_0(\alpha, \beta)$
-	<1	$(A-B)/2\alpha + (A+B)(X-Y)/2\pi\alpha$
-	>1	$(A-B)/2\alpha - (A+B)(X+Y)/2\pi\alpha$
+	<1	$(A-B)/2\alpha + (A+B)(X-Y)/2\pi\alpha$
+	>1	$(A-B)/2\alpha + (A+B)(X+Y)/2\pi\alpha$

Note: $X = \sin^{-1} |(\Delta + \alpha)/\beta|$

$Y = \sin^{-1} |(\Delta - \alpha)/\beta|$

The complete Fourier series analysis for determining $N_0(\alpha, \beta)$ and $N_1(\alpha, \beta)$ is given in chapter 2.

Returning to the system loops shown in figure 5, we express the input to the nonlinear block as

$$(1-3) \quad e(t) = \alpha + \beta \sin \omega t$$

where α = dc component in volts,
 β = peak value of the first harmonic
in volts,

and $\omega/2\pi$ = oscillation frequency in Hz.

Considering the system loops of figure 5, the balance condition for both the dc component and the fundamental component will be given by the following two simultaneous equations:

$$(1-4) \quad N_0(\alpha, \beta)G(0)D(0) - V_R'/\alpha = -1$$

$$(1-5) \quad N_1(\alpha, \beta)G_1(j\omega)D_1(j\omega) = -1$$

Here $G(0)$ and $D(0)$ are values of transfer functions $G(j\omega)$ and $D(j\omega)$ for the dc value where ω equals zero, and both $G(0)$ and $D(0)$ equal unity. Equations (1-4) and (1-5) can now be rewritten as

$$(1-6) \quad N_0(\alpha, \beta) - V_R'/\alpha = -1$$

$$(1-7) \quad G_1(j\omega)D_1(j\omega) = -1/N_1(\alpha, \beta)$$

Equation (1-6) is a scalar equation and (1-7) is a complex equation. The two are then actually three equations from which the three unknowns, namely, α, β , and ω can be determined.

It is difficult to solve the three simultaneous equations directly because of the arc-sine functions involved. Hence, the graphical approach suggested by Hsu and Meyer is utilized here with certain modifications. First the value of β is determined as a function of α from the equation (1-6). Then utilizing pairs of α and β , the function $-1/N_1(\alpha, \beta)$ is calculated and plotted in the complex plane. Also a Nyquist plot $G_1(j\omega)D_1(j\omega)$ is plotted in the same plane, with varying values of ω corresponding to different frequencies. Wherever the two curves intersect in the complex plane, the equation (1-7) is satisfied and hence it represents the operating point. The value of frequency corresponding to the intersection, is the frequency of oscillation.

The reason for lumping the delay function $D_1(j\omega)$ and the transfer function $G_1(j\omega)$, is obvious. Both are linear and dependent on frequency, and hence can be handled together as a function of ω .

An Example

As an example of applying the theory developed so far, consider the circuit shown in figure 7. This circuit is capable of providing a regulated dc output voltage of 10 volts, with an input voltage variation between 20 volts and 30 volts and a load variation from no load to 3.5 amps. The transistor (Q_2) is a fast switching device

which is driven by the voltage regulator (VR). Transistor (Q_1) forms an intermediate stage that provides the additional gain. The diode (D_1) is a fast switching device that provides an alternate path to the load current when transistor (Q_2) is turned off. Thus the transistor (Q_2) and the diode (D_1) together form the nonlinear circuit with an output changing from

$$A = (V_i - V_{Q_2}) \text{ volts}$$

to $-B$, the diode voltage drop.

Here V_{Q_2} = Saturation drop across Q_2 . In the circuit under consideration

$$V_{Q_2} = 0.8 \text{ V}$$

and $B = 1 \text{ V}$

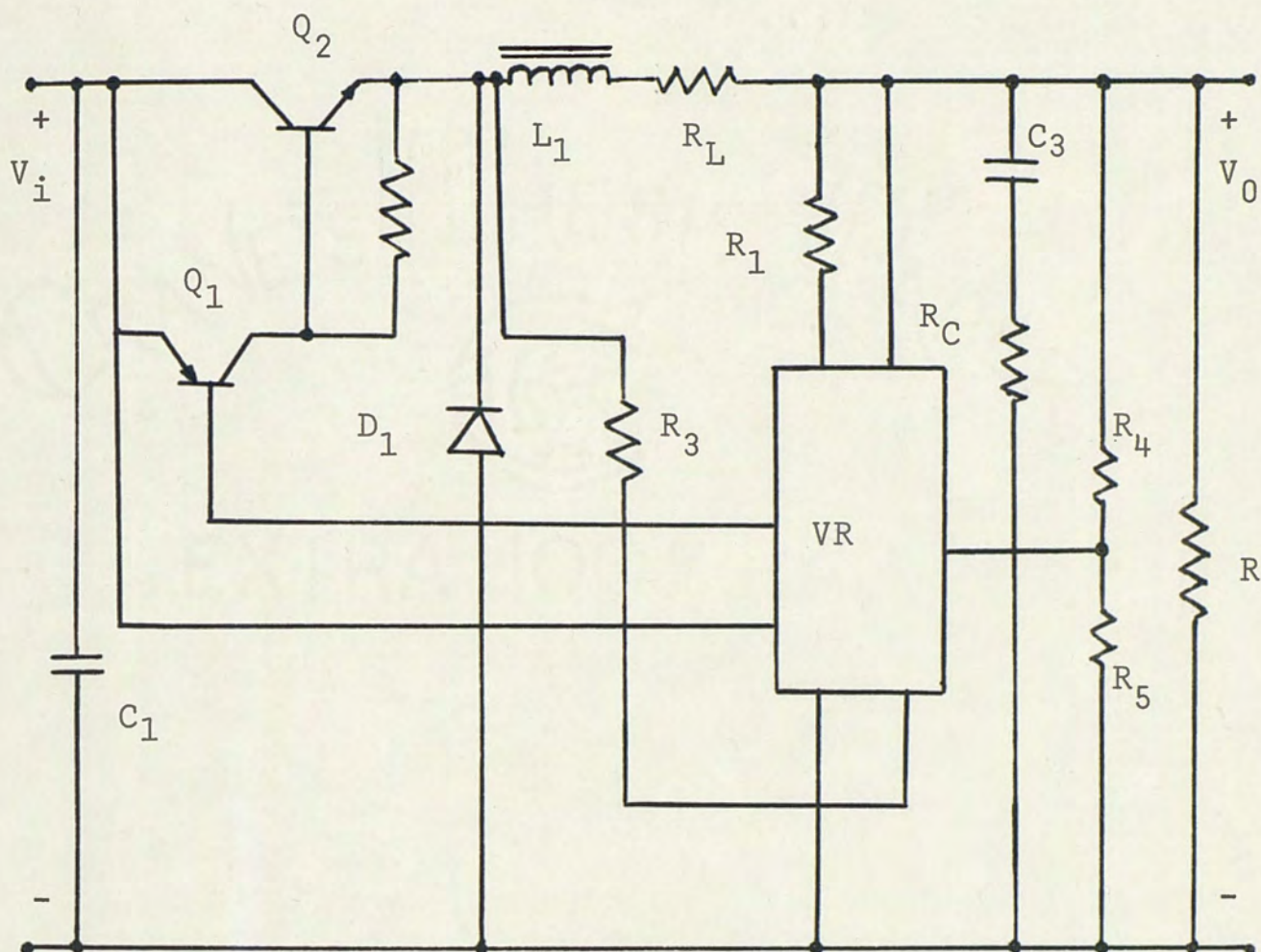
The hysteresis is provided by the resistor (R_3) and the switching voltage (Δ) is calculated as

$$0.07 \text{ volt for an input of 25 volts.}$$

Complete details of this calculation are shown in chapter 3.

The output filtering is provided by the inductor (L_1) and the capacitor (C_3). In the circuit model built for verification, inductor (L_1) has a series resistance of 0.096 ohms and the capacitor (C_3) has an effective series resistance of 0.379 ohms. The resistors R_4 and R_5 provide the necessary voltage division required for operation of the voltage regulator. The capacitor (C_1) is added

Figure 7



The Laboratory Model

L_1 : $530\mu\text{H}$, R_L : 0.096ohm

C_3 : $98\mu\text{F}$, R_C : 0.379ohm

R_3 : 1 megohm

VR: LM 300 voltage regulator

to minimize the effect of surges in the input voltage. Being on the input side the capacitor C_1 has no effect on the calculations and is not considered in the following analysis.

The time delay comprises of switching delays in transistors Q_1 and Q_2 and the delay in voltage regulator VR. The total time delay for the control system under consideration is estimated at 2.5 μ seconds. This is further discussed in chapter 3, where the various elements of the time delay are indicated.

Following the procedure suggested in the earlier section, values of β are first calculated as a function of α , using equation (1-6). To evaluate this, a set of values was assigned to β for a given value of α . Then for each value of α and β , the left hand side of the equation (1-6) was calculated and then the value of β was varied until the expression equalled -1. Knowing the value of α and $\beta(\alpha)$, the real and imaginary parts of function $-1/N_1(\alpha, \beta)$ are calculated using the equation (1-2).

Table 2 shows a set of values of α , $\beta(\alpha)$ and $-1/N_1(\alpha, \beta)$ for an input voltage of 25 volts. The expression $-1/N_1(\alpha, \beta)$ is shown in real and imaginary parts and is then plotted in the fourth quadrant of the complex plane as shown in figure 8.

Also shown in figure 8 is the Nyquist plot with the

TABLE 2

Values of $\beta(\alpha)$ and $-1/N_1(\alpha, \beta)$ for Varying α

α	β	$-1/N_1(\alpha, \beta)$	
		Real Part	Imaginary Part
-0.010	0.08854	-0.0333	-0.00453
-0.008	0.08273	-0.0266	-0.00452
-0.006	0.07793	-0.0200	-0.00453
-0.004	0.07442	-0.0137	-0.00453

Note:

$$\Delta = 0.07 \text{ V}$$

$$\text{Input} = 25 \text{ V} = V_i$$

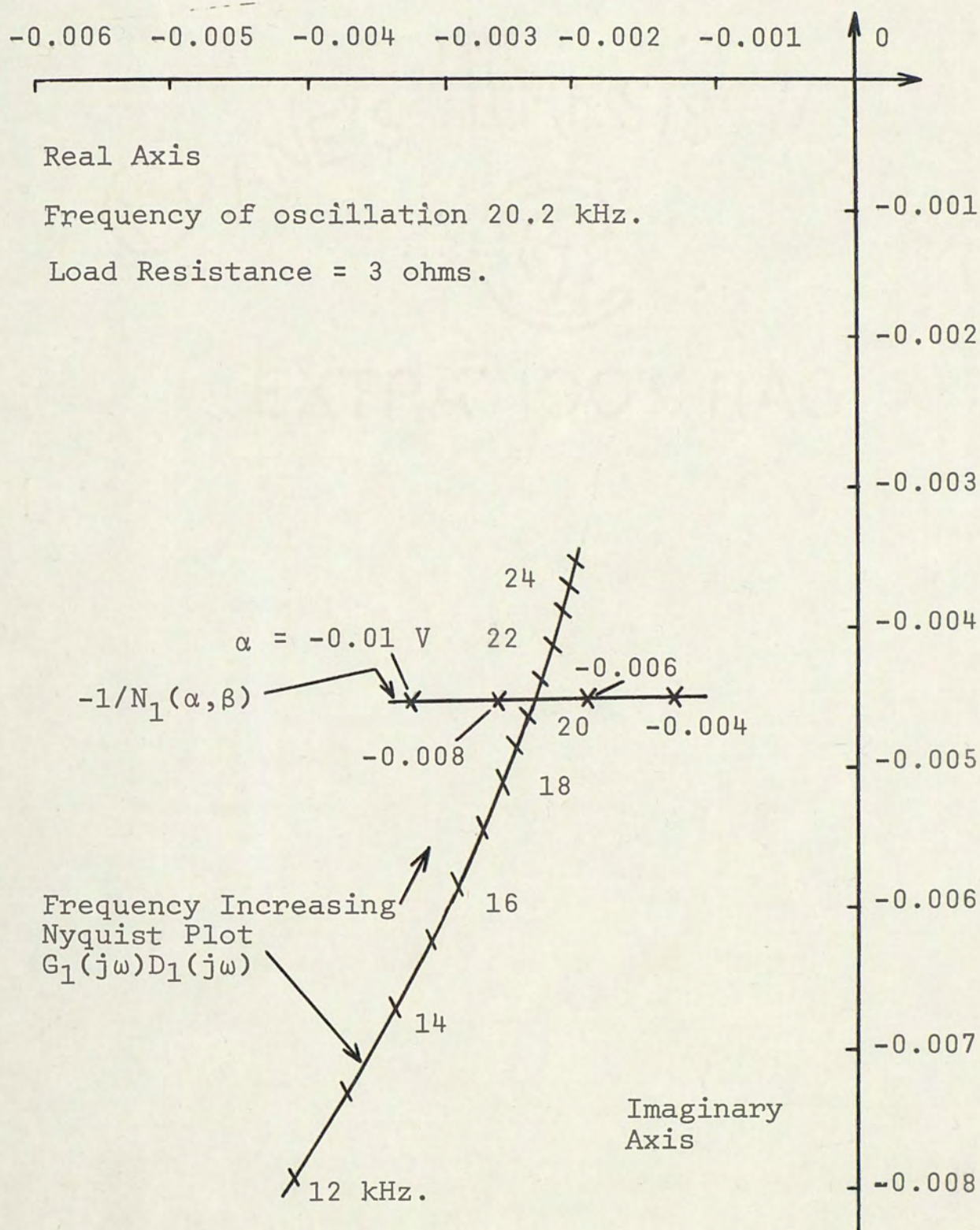
$$V_{Q2} = 0.8 \text{ V}$$

$$B = 1 \text{ V}$$

$$V'_R = 10.053 \text{ V}$$

$$A = 24.2 \text{ V}$$

Figure 8

The Graphical Solution

value of load resistance as 3 ohms. The two curves intersect at the frequency value of 20.2 kHz. Test results on the laboratory model showed a frequency of 21.7 kHz. Thus the test result agrees with the analytical result within close limits.

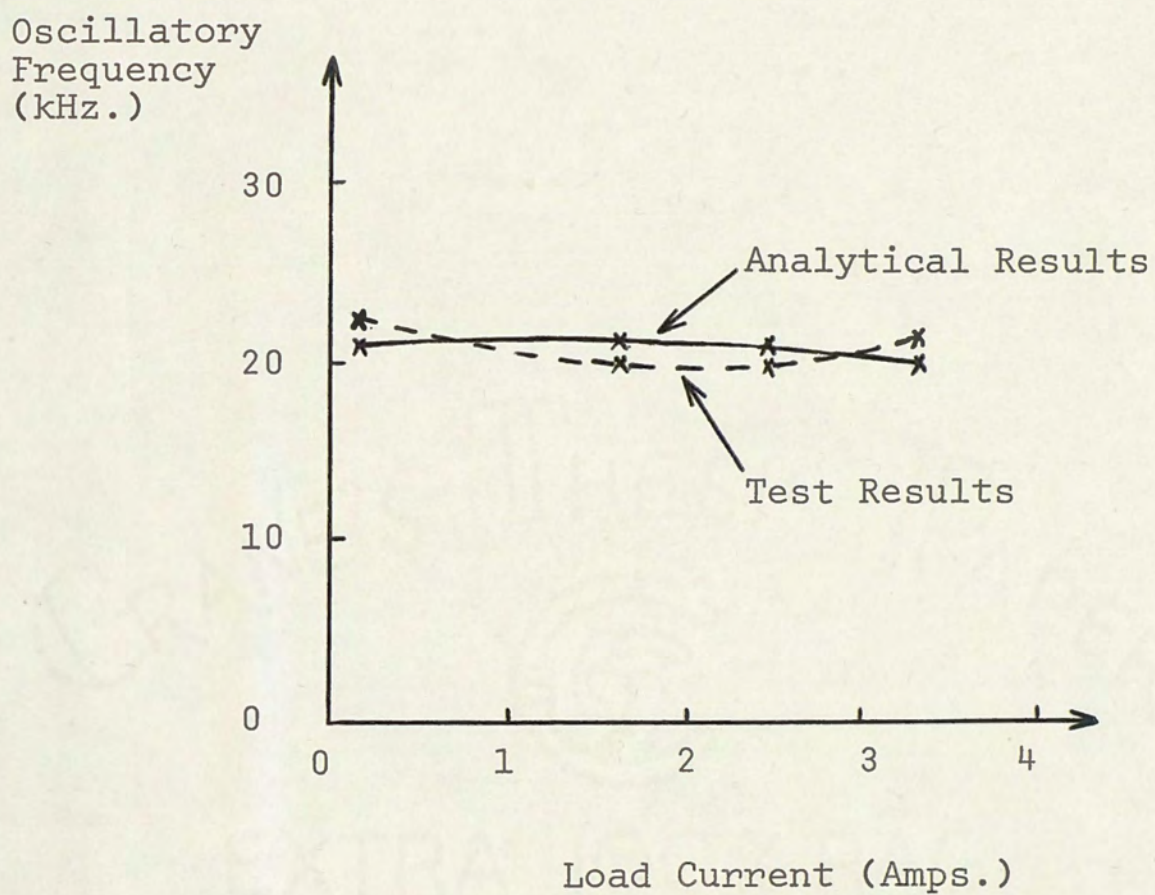
Figures 9 and 10 are graphs showing the variation of oscillatory frequency with changes in load and changes in input voltage respectively. The calculated values are shown by the heavy continuous lines while the test results are shown by the dotted lines. The calculations agree with the test values under all conditions of load and input voltage variations.

The Time Delay Function

The inclusion of a time delay function in the model improves the accuracy of calculations by bringing the model close to real life where such delays invariably exist. If the Nyquist plot is made with varying magnitudes of the time delay as shown in figure 11, the resulting changes in the oscillatory frequency are shown in table 3. As can be seen from the table, the time delay function has some effect on the calculated value of the frequency. However, this effect is of a small magnitude for the laboratory model under consideration.

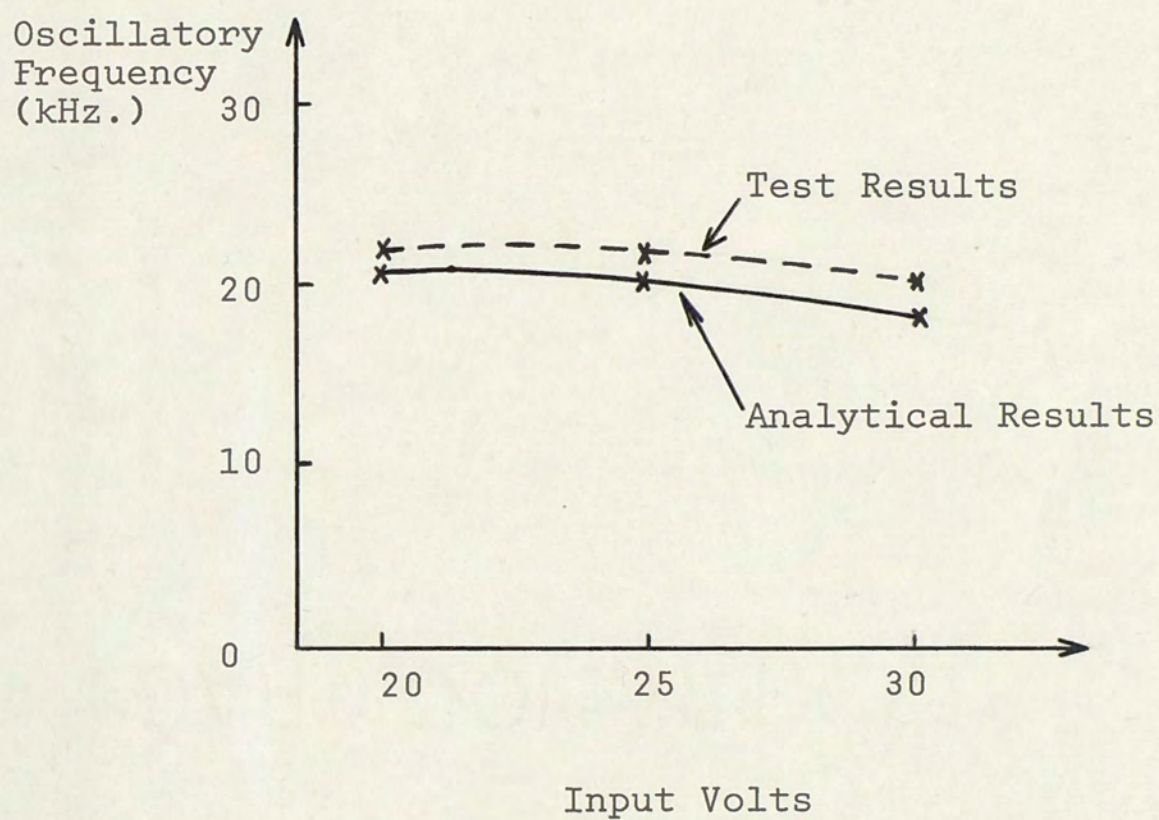
In the system model presented here, all the time delay elements are lumped together and the Nyquist plot is

Figure 9

Oscillatory Frequency versus Load Current

Note: Input Voltage 25 Volts.

Figure 10

Oscillatory Frequency versus Input Voltage

Note: Load Current 3.3 Amps.

Figure 11

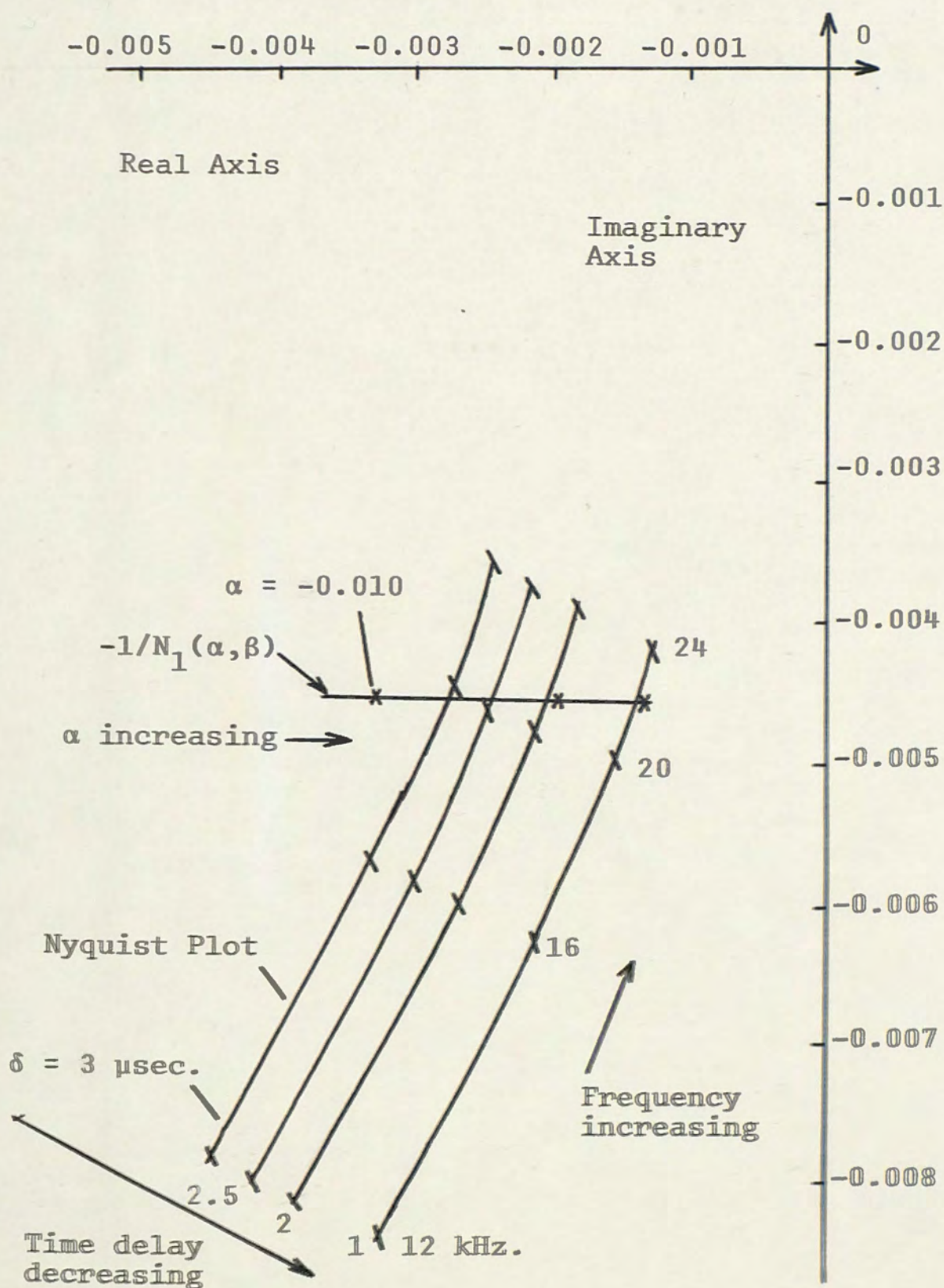
Effect of Varying Time Delay

TABLE 3

Change of Calculated Frequency with
Varying Time Delay

Time Delay (microseconds)	Calculated Frequency (kHz.)
3.0	19.7
2.5	20.2
2.0	20.9
1.0	21.8

Note:

Input Voltage 25 Volts.
Load Current 3.3 Amps.
Test Frequency 21.7 kHz.

modified to include the effect of various time delays occurring in the different sections of the circuit.

Other Implications

It was mentioned earlier that the analysis presented here provides a method of calculating oscillatory frequency. Various other performance characteristics are also involved in this analysis. These include α and β which represent the error in the dc output voltage and the magnitude of ripple voltage in the model respectively. The calculation of α and β at the intersection of the two curves does provide an idea as to how these important performance characteristics are related to the circuit parameters.

For example, in the graphical solution shown in figure 8, the values of α and β at the point of intersection are -0.0072 volt and 0.0772 volt respectively. This implies a peak to peak ripple of 0.154 volt as against the test value of 0.275 volt.

Considering the graphical solution in figure 8, a reduction in the switching voltage (Δ) will shift the curve for $-1/N_1(\alpha, \beta)$ closer to the real axis and shift the point of intersection with the Nyquist plot in such a way as to increase the oscillatory frequency. If the oscillatory frequency can not be increased beyond certain limits and this is the case most of the time, then some other cir-

cuit parameters must be changed to compensate the effect of reduced switching voltage (Δ). This can be accomplished by reducing the effective series resistance (R_C). The Nyquist plot will then shift in such a manner as to counter the effect of reduced switching voltage (Δ). Figure 12 shows how this is effected graphically. Of course, the same result could be obtained by increasing the magnitudes of inductance (L) or capacitance (C), which would result in a larger unit.

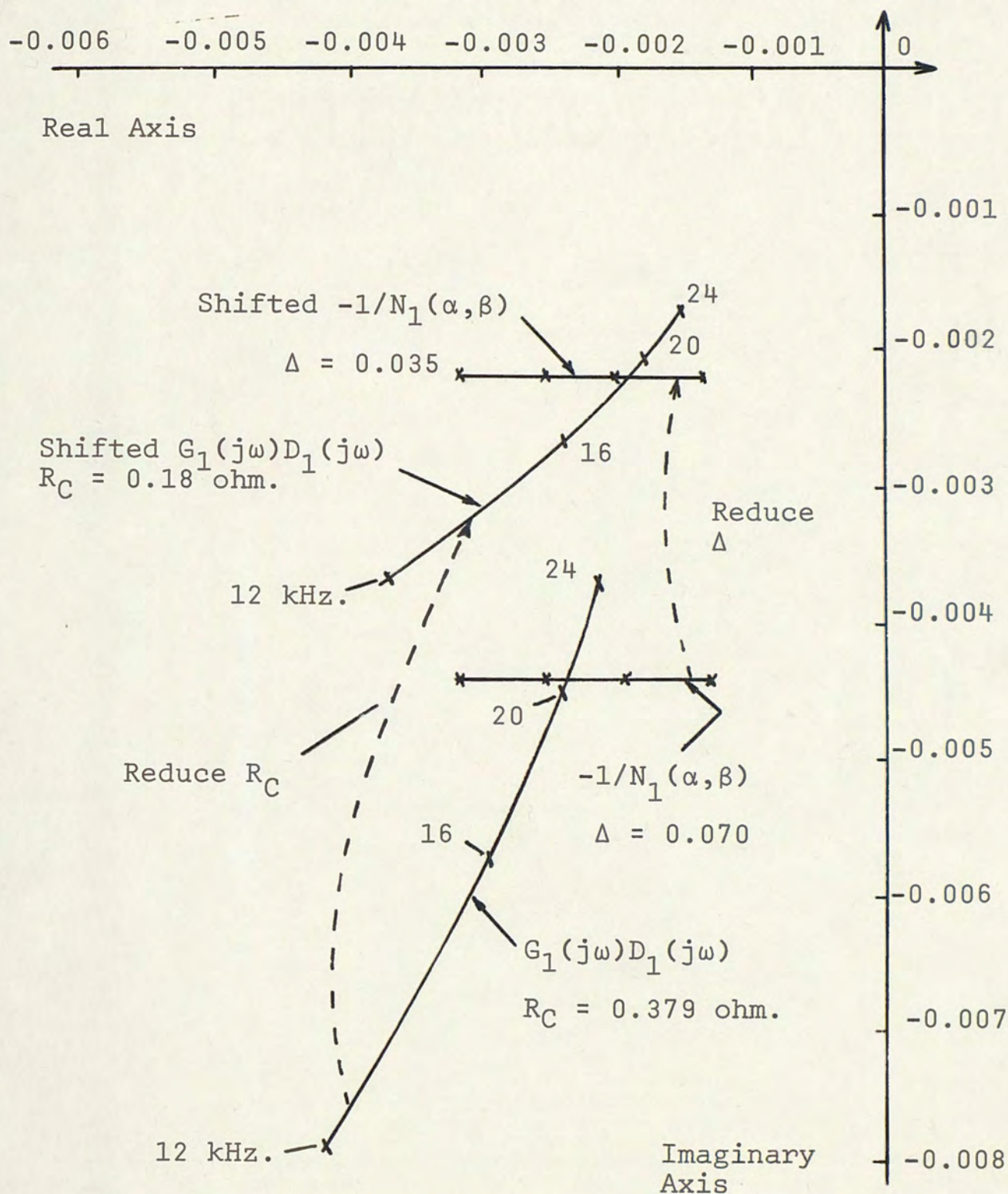
Remarks

In the past, Schweitzer and Rosenstein (1964), presented an approach using the describing function technique to determine the frequency of oscillation. The method presented by them had the basic limitation that it could apply only in one special case, that is when the value of α equals zero. This occurs when the switching on and off periods are equal. Clearly, such an approach is completely inadequate. Also no consideration was given to the time delay function.

More recently Judd and Chen (1972) presented an analytical approach termed "exact analysis". The term "exact analysis" was employed because the analysis considered all the harmonics of the ac ripple. However, when getting practical results, all the terms related to the harmonics were completely dropped out with the conten-

Figure 12

Effect of Reducing R_C and Δ



Frequency of oscillation- original curves: 20.2 kHz.

shifted curves: 18.8 kHz.

tion that it was adequate to use only the fundamental ac value. Here too, no consideration was given to the time delays involved.

Apart from the inclusion of time delay factor, this author also includes the voltage drop across switching transistor and the forward voltage drop across the diode, in his analysis. These are important factors particularly when dealing with outputs in low voltage range where the voltage drops become significant.

The procedure suggested by this author should be useful when designing switching regulated dc power supplies. When a preliminary design based on nominal input and output requirements is completed, this procedure can be applied to calculate the oscillatory frequencies over the entire range of input voltage and load current. The designer can then verify if the circuit components such as the switching transistor, driver circuit, the filter inductor, and the capacitor are suitable for operation over the entire range of frequency.

If some of the components are not suitable, selecting appropriate components may be necessary. On the other hand, it may be possible to alter some of the characteristics of the circuit so that the operating frequency is shifted to a more desirable range. For example, increasing the switching voltage (Δ) will reduce the operating frequency. If a filter capacitor

with a lower series resistance can be substituted, it will be possible to increase the operating frequency. The same result could be obtained by selecting the switching transistor or the driver circuit with shorter time delay.

Thus the method suggested here provides a tool for the designer in selecting circuit components and predicting steady state performance of the switching regulated dc power supply.

CHAPTER 2

THE MATHEMATICAL MODEL

In the last chapter, certain functions were used for representing various sections of the control system. The author presents below a complete derivation of these functions.

The Nonlinear Section

The hysteresis loop shown in figure 6 produces a rectangular output having dc and ac components. To calculate the dc describing function and the first harmonic describing function, the amount of dc output voltage (V_D) and the peak value of the first harmonic (V_F) must be determined. Then

$$(2-1) \quad N_0(\alpha, \beta) = V_D/\alpha \text{ and}$$

$$(2-2) \quad N_1(\alpha, \beta) = V_F/\beta$$

V_D and V_F are calculated by Fourier series analysis, using figure 6, as follows:

$$(2-3) \quad V_D = (1/2\pi) \left\{ \int_{\omega t_1}^{\omega t_2} A d\omega t - \int_0^{\omega t_1} B d\omega t - \int_{\omega t_2}^{2\pi} B d\omega t \right\}, \text{ where}$$

$$(2-4) \quad \sin \omega t_1 = (\Delta - \alpha)/\beta, \quad \omega t_1 = \sin^{-1} |(\Delta - \alpha)/\beta|$$

$$(2-5) \quad \sin \omega t_2 = -(\Delta + \alpha)/\beta, \quad \omega t_2 = \pi + \sin^{-1} |(\Delta + \alpha)/\beta|$$

From these equations:

$$(2-6) \quad V_D = \{A(\omega t_2 - \omega t_1) + B(\omega t_2 - \omega t_1) - 2\pi B\} / 2\pi, \text{ or}$$

$$(2-7) \quad V_D = (A-B)/2 +$$

$$(A+B)\{\sin^{-1}|(\Delta+\alpha)/\beta| - \sin^{-1}|(\Delta-\alpha)/\beta|\} / 2\pi$$

The ac fundamental component (V_F) can be expressed as

$$(2-8) \quad P+jQ = \{ \int_0^{2\pi} u \sin \omega t d\omega t + j \int_0^{2\pi} u \cos \omega t d\omega t \} / \pi,$$

where P is the in-phase component and Q is the quadrature component. The term u equals A or -B.

Now,

$$\begin{aligned} P &= \{ - \int_0^{\omega t_1} B \sin \omega t d\omega t + \int_{\omega t_1}^{\omega t_2} A \sin \omega t d\omega t - \int_{\omega t_2}^{2\pi} B \sin \omega t d\omega t \} / \pi \\ &= \{ B \cos \omega t \Big|_0^{\omega t_1} - A \cos \omega t \Big|_{\omega t_1}^{\omega t_2} + B \cos \omega t \Big|_{\omega t_2}^{2\pi} \} / \pi \\ &= (A \cos \omega t_1 - A \cos \omega t_2 + B \cos \omega t_1 - B \cos \omega t_2) / \pi \end{aligned}$$

Therefore,

$$(2-9) \quad P = (A+B)(\cos \omega t_1 - \cos \omega t_2) / \pi$$

From equations (2-4) and (2-5),

$$(2-10) \quad \cos \omega t_1 = \{1 - (\Delta - \alpha)^2 / \beta^2\}^{1/2}, \text{ and}$$

$$(2-11) \quad \cos \omega t_2 = -\{1 - (\Delta + \alpha)^2 / \beta^2\}^{1/2}$$

Using equations (2-9), (2-10) and (2-11), we get

$$(2-12) \quad P = \frac{(A+B)}{\pi} \left[\{1-(\Delta-\alpha)^2/\beta^2\}^{\frac{1}{2}} + \{1-(\Delta+\alpha)^2/\beta^2\}^{\frac{1}{2}} \right]$$

Now,

$$\begin{aligned} Q &= \left\{ - \int_0^{\omega t_1} B \cos \omega t d\omega t + \int_{\omega t_1}^{\omega t_2} A \cos \omega t d\omega t - \int_{\omega t_2}^{2\pi} B \cos \omega t d\omega t \right\} / \pi \\ &= \left\{ -B \sin \omega t \Big|_0^{\omega t_1} + A \sin \omega t \Big|_{\omega t_1}^{\omega t_2} - B \sin \omega t \Big|_{\omega t_2}^{2\pi} \right\} / \pi \end{aligned}$$

Therefore,

$$(2-13) \quad Q = (A+B)(\sin \omega t_2 - \sin \omega t_1) / \pi$$

From equations (2-4) and (2-5)

$$\begin{aligned} \sin \omega t_2 &= \sin \{ \pi + \sin^{-1} |(\Delta+\alpha)/\beta| \} \\ &= -|(\Delta+\alpha)/\beta|, \end{aligned}$$

$$\text{and } \sin \omega t_1 = |(\Delta-\alpha)/\beta|,$$

Because Δ, α and β are all positive,

$$(2-14) \quad \sin \omega t_2 = -(\Delta+\alpha)/\beta, \quad \text{and}$$

$$(2-15) \quad \sin \omega t_1 = (\Delta-\alpha)/\beta.$$

From equations (2-13), (2-14) and (2-15), we get

$$(2-16) \quad Q = -2\Delta(A+B)/\pi\beta$$

Therefore, the ac fundamental component is given by

$$\begin{aligned} (2-17) \quad V_F &= P + jQ \\ &= \frac{A+B}{\pi} \left[\{1-(\Delta-\alpha)^2/\beta^2\}^{\frac{1}{2}} + \{1-(\Delta+\alpha)^2/\beta^2\}^{\frac{1}{2}} - j2\Delta/\beta \right] \end{aligned}$$

From equations (2-1), (2-2), (2-7) and (2-17), we get the following results:

$$(2-18) \quad N_0(\alpha, \beta) = (A-B)/2\alpha +$$

$$(A+B)\{\sin^{-1}|(\Delta+\alpha)/\beta| - \sin^{-1}|(\Delta-\alpha)/\beta|\}/2\pi\alpha$$

$$(2-19) \quad N_1(\alpha, \beta) = \frac{(A+B)}{\pi\beta} \left[\{1-(\Delta-\alpha)^2/\beta^2\}^{\frac{1}{2}} + \{1-(\Delta+\alpha)^2/\beta^2\}^{\frac{1}{2}} - j2\Delta/\beta \right]$$

So far we have considered α to be positive and less than Δ . This is not always true. Four distinct cases arise as α is allowed to vary. The switching angles shift from one quadrant to another as α changes from positive to negative and also as the magnitude of α exceeds the value of Δ . Values of $N_0(\alpha, \beta)$ under various conditions have already been shown in table 1 of chapter 1. The transfer function $N_1(\alpha, \beta)$ does not change and remains at the value given by the equation (2-19).

The Linear Section

The transfer function of the linear portion of the circuit is calculated in the following manner. Figure 13 shows all the elements of this circuit. These are:

- L = The value of inductance in henrys
- R_L = Effective resistance of the inductor in ohms
- C = The value of capacitance in farads
- R_C = Effective series resistance of the capacitor in ohms
- R = Load resistance in ohms

Figure 13

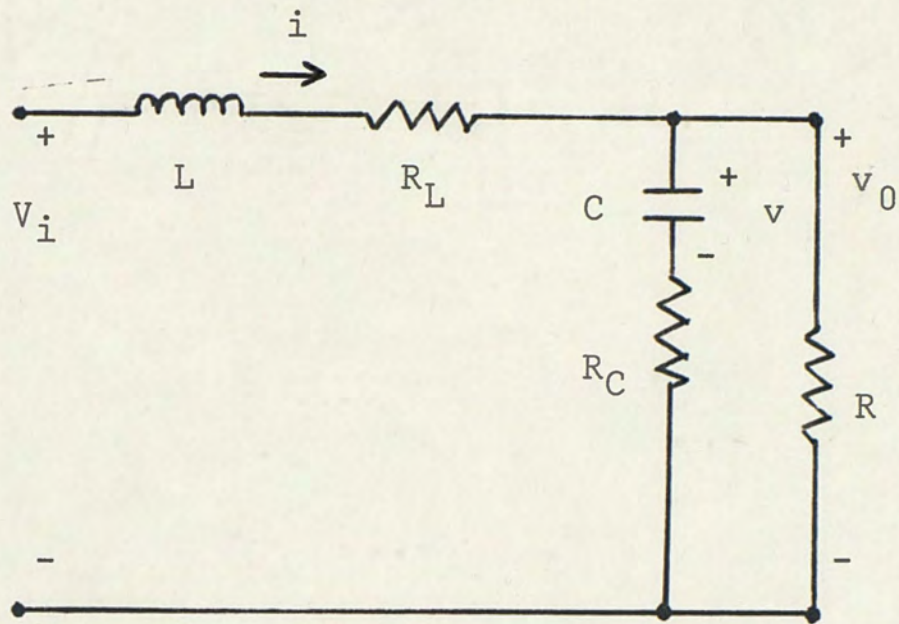
The Linear Section

Figure 14

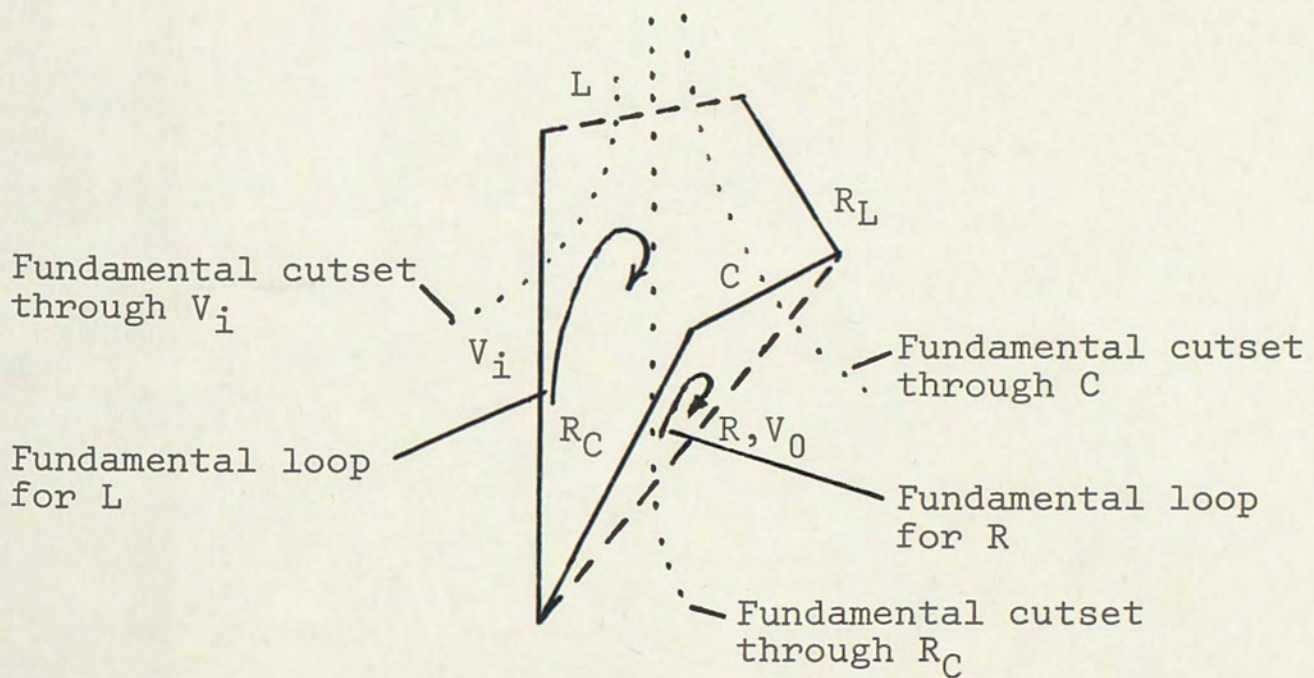
The Normal Tree for Linear Section

Figure 14 shows the normal tree for the circuit. We are interested in the voltage transfer function $G(s)$, such that

$$(2-20) \quad v_0(s) = G(s)V_i(s),$$

where $v_0(s)$ = Laplace transform of the output voltage,
and $V_i(s)$ = Laplace transform of the input voltage.

Using the standard form state equations, the state of the circuit can be expressed as

$$(2-21) \quad \dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}u$$

and

$$(2-22) \quad y = \underline{C}\underline{x} + \underline{D}u$$

where $\underline{x} = \begin{bmatrix} i \\ v \end{bmatrix},$

i and v being the inductor current and capacitor voltage respectively, as shown in figure 13.

The input $u = V_i$,

and the output $y = v_0$

From the normal tree of figure 14, we get

$$(2-23) \quad V_i = Li + R_L i + v + C\dot{v}R_C$$

$$(2-24) \quad i = C\dot{v} + V_0/R \text{ and}$$

$$(2-25) \quad v_0 = v + C\dot{v}R_C$$

Rearranging the above, we get

$$(2-26) \quad \dot{i} = - \frac{(RR_L + RR_C + R_C R_L)i}{L(R+R_C)} - \frac{Rv}{L(R+R_C)} + \frac{V_i}{L},$$

$$(2-27) \quad \dot{v} = \frac{Ri}{C(R+R_C)} - \frac{v}{C(R+R_C)}, \text{ and}$$

$$(2-28) \quad v_0 = \frac{RR_C i}{R+R_C} + \frac{Rv}{R+R_C}$$

Comparing equations (2-21), (2-22) with (2-26), (2-27) and (2-28), we get the following:

$$\underline{A} = \begin{bmatrix} -\frac{RR_L + RR_C + R_C R_L}{L(R+R_C)} & -\frac{R}{L(R+R_C)} \\ \frac{R}{C(R+R_C)} & -\frac{1}{C(R+R_C)} \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} \frac{RR_C}{R+R_C} & \frac{R}{R+R_C} \end{bmatrix}$$

$$\underline{D} = 0$$

The voltage transfer function is given by

$$(2-29) \quad G(s) = \underline{C} [s\underline{I} - \underline{A}]^{-1} \underline{B}$$

Solution of the above yields

$$(2-30) \quad G(s) = \frac{sCRR_C + R}{s^2 LC(R+R_C) + s\{L + CRR_C + CR_L(R+R_C)\} + (R+R_L)}$$

The Nyquist Plot

In chapter 1, the Nyquist plot was drawn in the complex plane from the product $G_1(j\omega)D_1(j\omega)$. $G_1(j\omega)$ can be determined from equation (2-30) by substituting $j\omega$ for s . $D_1(j\omega)$ is the time delay function expressed as $e^{-j\omega\delta}$. This can also be written as a complex number:

$$(2-31) \quad D_1(j\omega) = \cos\omega\delta - j\sin\omega\delta$$

Using equations (2-30) and (2-31), the function $G_1(j\omega)D_1(j\omega)$ can be evaluated for a given value of frequency.

CHAPTER 3

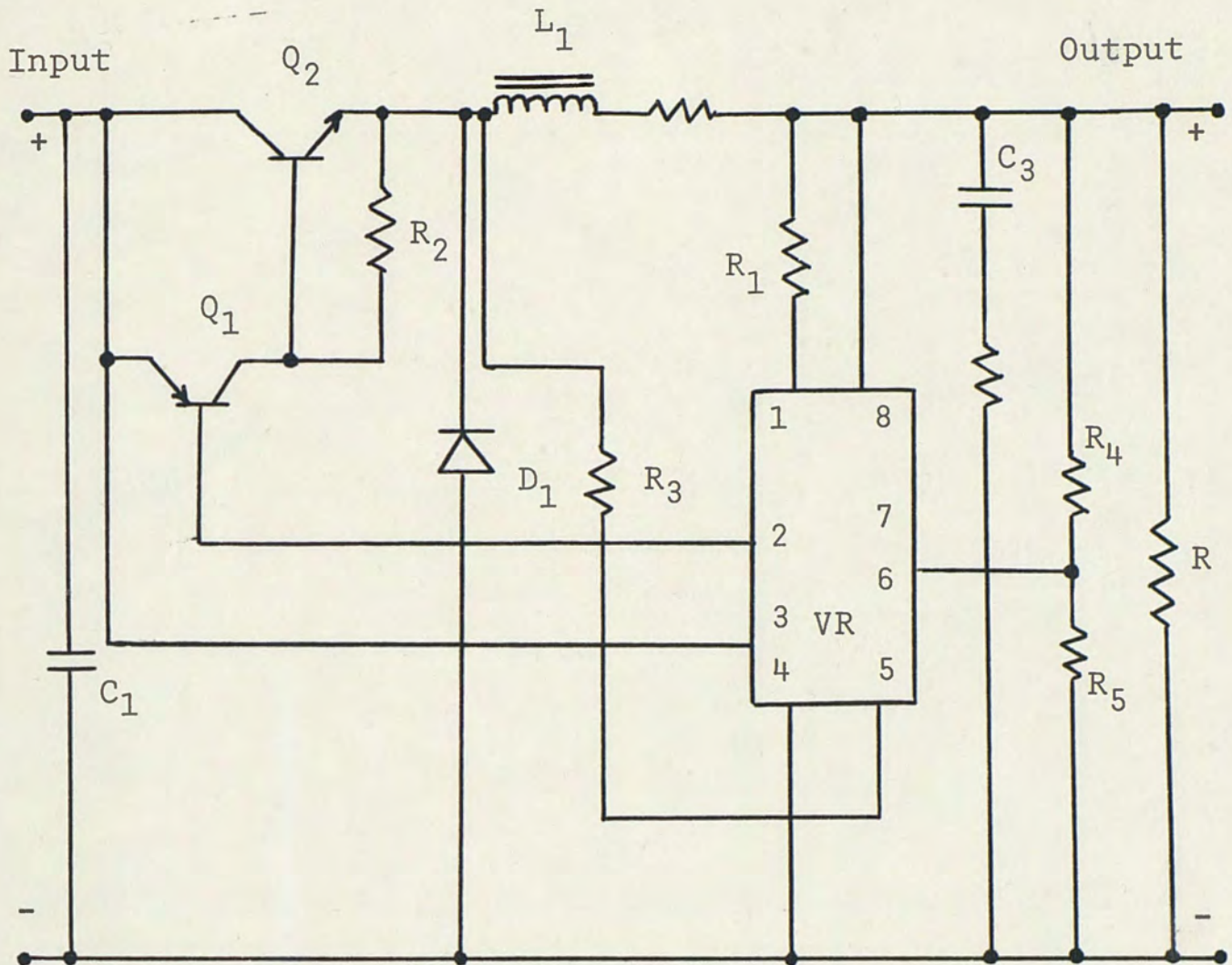
THE LABORATORY MODEL AND ITS CHARACTERISTICS

The circuit shown in figure 7 was briefly discussed by the author. Figure 15 show the components used in this circuit. The voltage regulator is a type LM 300 of National Semiconductor Corporation. The complete working of the regulator and the circuit is explained in the application note written by Widlar (1969). Only a brief description related to the present analysis is given below.

The LM 300 is an integrated circuit, linear voltage regulator. Figure 16 shows its circuit arrangement in a block diagram. When an unregulated dc voltage is applied to the terminal 3, a temperature compensated reference output of 1.8 volts is produced at the terminal 5. This output is fed to a differential amplifier that compares the feedback voltage connected to terminal 6, with the voltage at terminal 5. It provides an amplified output that drives another amplifier which in turn provides the output that appears between terminals 2 and 3.

In the power supply circuit as shown in figure 15 (and also figure 7), the output voltage at the terminals

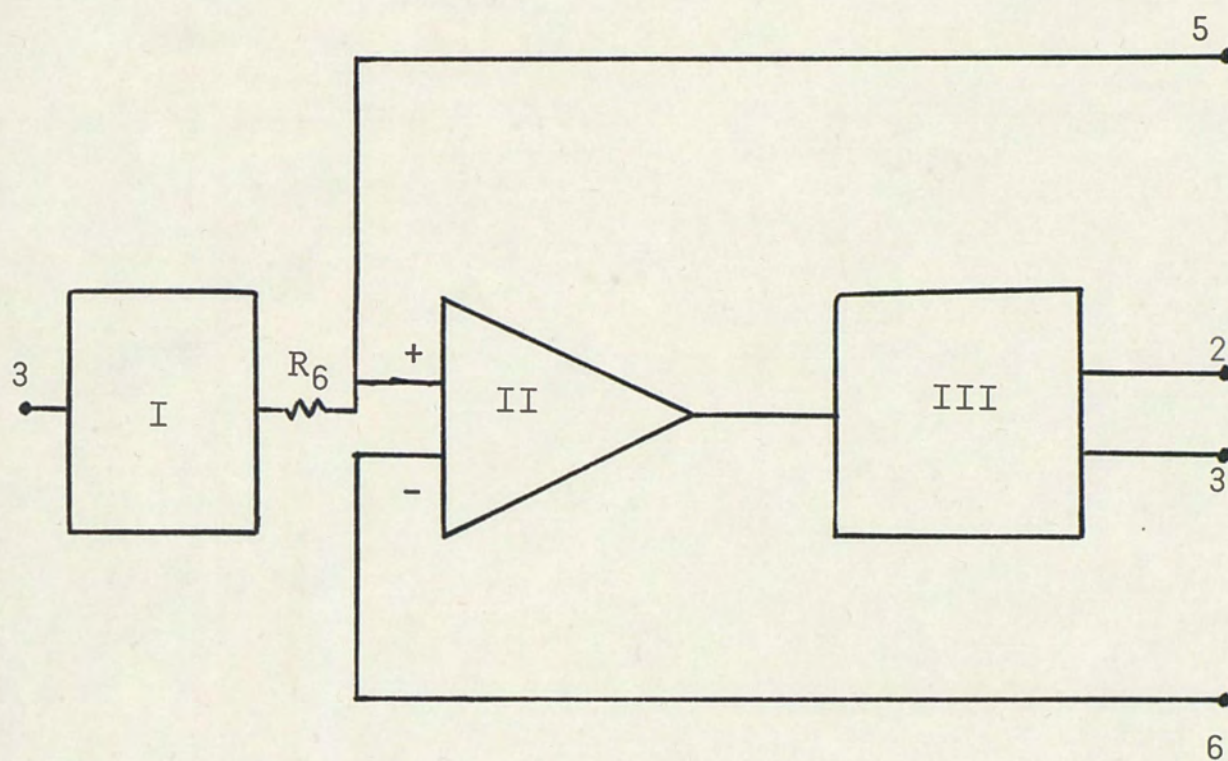
Figure 15

The Laboratory Model and Its Components(Q₂) - 2N3447(L₁) - 530 μH, 0.96 Ω(Q₁) - 2N2905A(R₁) - 22 Ω(D₁) - 1N3880(R₂) - 68 Ω

(VR) - LM300

(R₃) - 1 MΩ(C₁) - 22 μF,(R₄) - 11.1 kΩ(C₃) - 100 μF, 0.379 Ω(R₅) - 2.44 kΩ

Figure 16

LM 300 Voltage Regulator

- I Reference voltage generator
- II Differential amplifier
- III Amplifier

2 and 3 is connected to the emitter and base of the transistor (Q_1). This transistor is driven into saturation as soon as the required voltage is provided between the terminals 2 and 3. This occurs when the feedback voltage at the terminal 6 is lower than the reference voltage. When the transistor (Q_1) is switched on, a current flows through resistor R_2 which in turn causes the transistor (Q_2) to be driven into saturation. The transistor (Q_1) is used only because the voltage regulator is incapable of providing the necessary current to drive the larger transistor (Q_2). Both Q_1 and Q_2 are fast switching transistors. The diode (D_1) that provides the alternate path to the load current, is also a fast switching device.

The resistors R_4 and R_5 act as a voltage divider so that only a portion of the output voltage is sensed at the feedback terminal 6.

The Hysteresis

The hysteresis is obtained by connecting the terminal 5 of the voltage regulator to the output of the transistor (Q_2) via resistor R_3 as shown in figure 15. Referring to figure 16, R_6 is the internal resistance seen at terminal 5. This resistance causes the voltage at terminal 5 to change from its nominal value of 1.8 volts by an amount given by

$$(3-1) \quad h_1 = R_6 A / (R_3 + R_6)$$

when the transistor (Q_2) is in the on state. When the transistor (Q_2) is in the off state, the change is given by

$$(3-2) \quad -h_2 = -R_6 B / (R_3 + R_6)$$

In the equations (3-1) and (3-2), A and B are equal to $(V_i - V_{Q_2})$ and the diode drop (B) respectively. For the laboratory model under consideration V_{Q_2} is 0.8 volt, B is 1 volt and R_3 is 1 megohm.

From the manufacturer's information R_6 equals 1000 ohms.

Now, the voltage deviations h_1 and $-h_2$ are referred to a fraction of the output voltage. To calculate the switching voltage (Δ), we take the average value of the deviation and multiply the amount by the ratio $(R_4 + R_5)/R_5$, where R_4 and R_5 form the voltage divider shown in figure 15. Then

$$(3-3) \quad \Delta = (h_1 + h_2)(13.54/2.44)/2$$

For an input voltage (V_i) equal to 25 volts, A equals 24.2 volts.

Then from equations (3-1), (3-2), and (3-3)

$$h_1 = 0.0242 \text{ volt}$$

$$h_2 = 0.001 \text{ volt}$$

and
$$\Delta = 0.07 \text{ volt}$$

It may be noted that the nominal output from the voltage regulator is shifted from 1.8 volt to $1.8 + (h_1 - h_2)/2$ and the nominal regulated output voltage is given by

$$V'_R = (13.54/2.44)\{1.8 + (h_1 - h_2)/2\} = 10.053.$$

The magnitude of switching voltage (Δ) changes as the input

voltage is changed for the laboratory model under consideration. It can be calculated for other operating input voltages, in a manner similar to the above.

The Time Delay

The laboratory model has three elements in which the time delays occur. These elements are: the transistor (Q_1), the transistor (Q_2), and the voltage regulator (VR) as shown in figure 16.

The transistor (Q_1) is a 2N2905A, pnp silicon transistor. The switching characteristics of this transistor are specified as follows:

Turn-On Time

Delay time :	0.006 microsec.
Rise time :	0.020 microsec.
Total turn-on time :	0.026 microsec.

Turn-Off Time

Storage time :	0.050 microsec.
Fall time :	0.020 microsec.
Total turn-off time :	0.070 microsec.

The delay time (δ_{Q_1}) for the above transistor is taken as an average of the turn-on time and the turn-off time.

Thus

$$\delta_{Q_1} = 0.048 \text{ microseconds.}$$

The transistor (Q_2) is a 2N3447, npn silicon power

transistor. The switching characteristics are given in a graphical form, from which the following average values are estimated:

Turn-On Time

Delay time :	0.13 microsec.
Rise time :	0.37 microsec.
Total turn-on time :	0.50 microsec.

Turn-Off Time

Storage time :	1.50 microsec.
Fall time :	0.37 microsec.
Total turn-off time :	1.87 microsec.

The delay time (δ_{Q_2}) for the transistor (Q_2) is taken as an average of the turn-on time and the turn-off time. Thus

$$\delta_{Q_2} = 1.19 \text{ microseconds.}$$

The voltage regulator (VR) is an integrated circuit device. The manufacturer's data for this device shows a total transient load response duration of 2.5 microseconds. The time delay (δ_{VR}) is estimated at half the transient load duration. Thus

$$\delta_{VR} = 1.25 \text{ microseconds.}$$

From the three individual elements of time delay, the total lumped delay (δ) is calculated as the sum of these three elements. Thus

$$\begin{aligned} \delta &= \delta_{Q_1} + \delta_{Q_2} + \delta_{VR} \\ &= 2.488 \text{ microseconds} \end{aligned}$$

\approx 2.5 microseconds.

This value of 2.5 microseconds has been used for the analysis of the laboratory model.

CHAPTER 4

COMPARISON AND CONCLUSIONS

The test values and the calculated values of oscillatory frequency for the laboratory model of figure 15 are shown in tables 4 and 5. The graphs shown in figure 9 and 10 were constructed from these tables. A comparison of the test values and calculations shows consistency over a wide range of operating conditions.

The author believes that the technique presented here will be of use when designing switching regulated, dc to dc power supplies. This has already been explained in the concluding part of chapter 1. The technique of the dual-input describing function can be extended to other power supplies where different types of filter circuits are employed. Another interesting field for application is the ac to dc power supply where a transformer operates at the high switching frequency, thereby reducing the overall size of the power supply. The technique developed here, can be extended to the analysis of these systems, by developing system model in a manner similar to the one adopted by the author.

When the laboratory model was tested at a low input

TABLE 4

Comparison of Calculations and Test Results
Varying Input Voltage

Input Voltage (V)	Frequency Test (kHz.)	Frequency Calculation (kHz.)
20	21.8	20.4
25	21.7	20.2
30	20.0	18.0

Note:

Load Current 3.3 Amps.

TABLE 5

Comparison of Calculations and Test Results
Varying Load Current

Load Current (A)	Frequency Test (kHz.)	Frequency Calculation (kHz.)
0.10	22.7	22.0
1.67	19.6	21.0
2.50	20.0	21.0
3.33	21.7	20.2

Note:

Input Voltage 25 Volts.

voltage of 15 volts, the circuit operated satisfactorily. However, the corresponding analysis of the model showed that the two curves $G_1(j\omega)D_1(j\omega)$ and $-1/N_1(\alpha, \beta)$ do not intersect. Similar results were also observed at a high input voltage of 35 volts. The low and high voltage operation of the circuit can not be analysed by the present model. The author believes this to be an area where further analytical work will be fruitful.

LIST OF REFERENCES

- Hsu, J. C. and Meyer, A. U.
Modern Control Principles and Applications.
New York: McGraw-Hill Book Company, 1968, p. 244.
- Judd, F. F. and Chen, C. T.
"On the Performance and Design of Self
Oscillating DC to DC Converters".
IEEE Transactions on Industrial Electronics
and Control Instrumentation, Vol. IECI-19, No. 3,
(August, 1972), pp. 89-97.
- Schweitzer, B. P. and Rosestein, A. B.
"Free Running Switching Mode Power Regulator:
Analysis and Design".
IEEE Transactions Aerospace, Vol. AS-2,
(October, 1964), pp. 1170-1180.
- Widlar, R. J.
Designing Switching Regulators.
Santa Clara: National Semiconductor Corporation,
Application Note AN-2, 1969.